CHAPTER 5

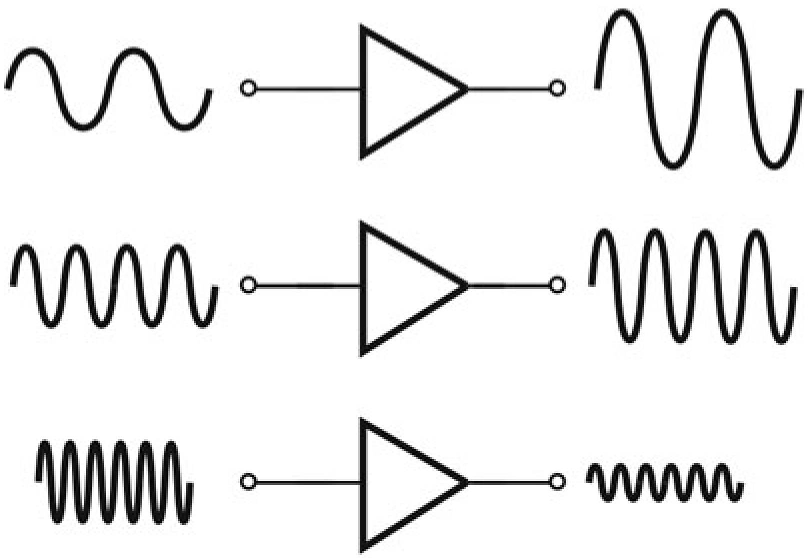
Frequency Response

Syllabus

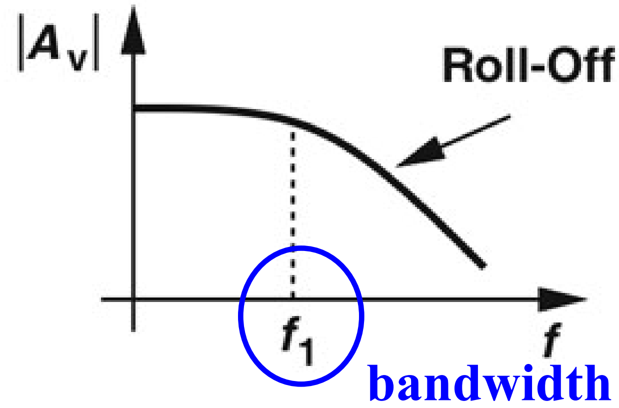
- Basic physics of MOS transistors and their equivalent circuit models (ch. 6)
- Elementary gain stages (ch. 7, ch. 9, and ch. 10)
- Operational amplifier basics (ch. 8)
- Frequency responses (ch. 11)
- Noise (ch.7, Design of analog CMOS ICs)
- Feedback, stability and compensation (ch. 12)
- Operational amplifiers (ch.9, Design of analog CMOS ICs)

What is “Frequency Response”

Apply a sinusoid at the input of the circuit and observe the output while the input frequency is varied.



Gain roll-off with frequency

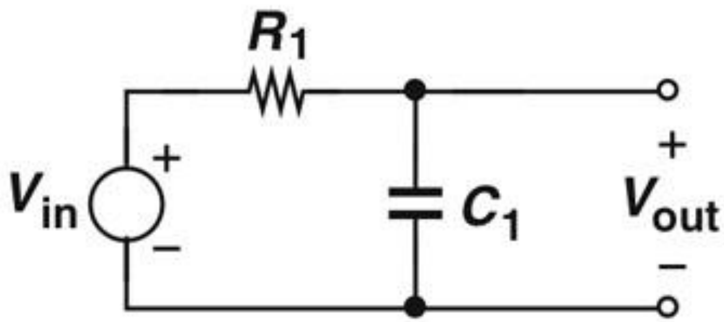


What is the cause??

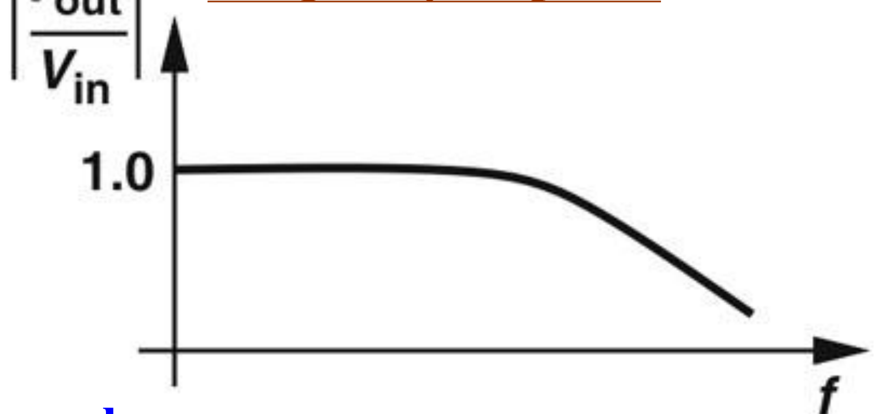
What Causes the Gain Roll-Off?

Take the low-pass filter for example

The circuit

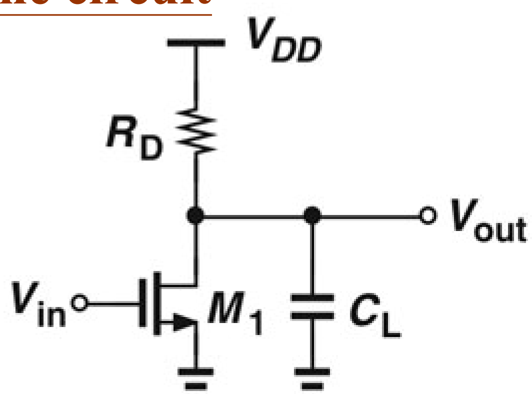


Frequency response

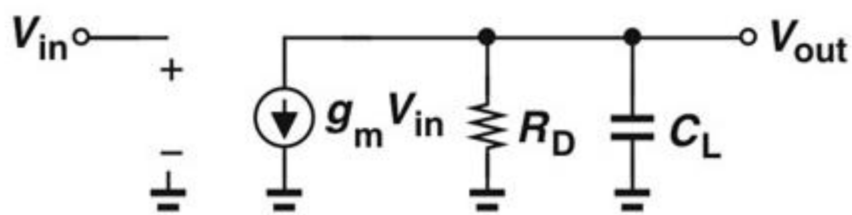


Take the common-source amplifier for example

The circuit



Small-signal model



$$V_{out} = -g_m V_{in} (R_D \parallel \frac{1}{C_L s}).$$

Gain drops at high frequencies!!

Transfer Function

From basic circuit theory, the transfer function of a circuit can be written as:

$$H(s) = A_0 \frac{\left(1+\frac{s}{\omega_{z1}}\right)\left(1+\frac{s}{\omega_{z2}}\right)\cdots}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)\cdots}, \qquad s \to 0 \qquad H(s) \to A_0$$

A_0**: the low-frequency gain**

ω_z**: zeros of the transfer function**

ω_p**: poles of the transfer function**

For input: $x(t) = A\cos(2\pi ft) = A\cos\omega t$

Output: $y(t) = A\,|H(j\omega)|\cos\left[\omega t + \angle H(j\omega)\right],$

$|H(j\omega)|$**: magnitude of the transfer function**

$\angle H(j\omega)$**: phase of the transfer function**

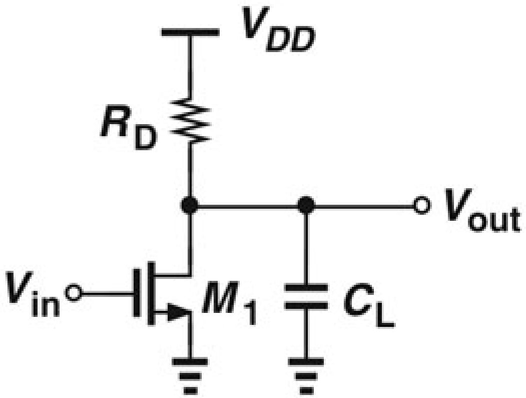
f**: in Hz**

ω **: in rad/s**

$\omega = 2\pi f$

Example 11.4

Determine the transfer function and frequency response of the CS stage



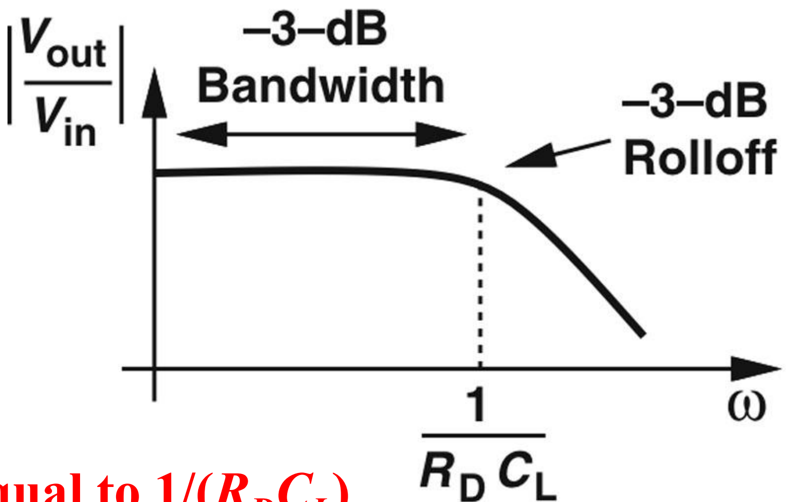
$$H(s) = \frac{V_{out}}{V_{in}}(s) = -g_m(R_D \parallel \frac{1}{C_L s})$$
$$= \frac{-g_m R_D}{R_D C_L s + 1}.$$

For sinusoid input: $s = j\omega$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{g_m R_D}{\sqrt{R_D^2 C_L^2 \omega^2 + 1}}.$$

At $\omega = 1/(R_D C_L)$ $\quad \left|\frac{V_{out}}{V_{in}}\right| = \frac{g_m R_D}{\sqrt{2}}.$

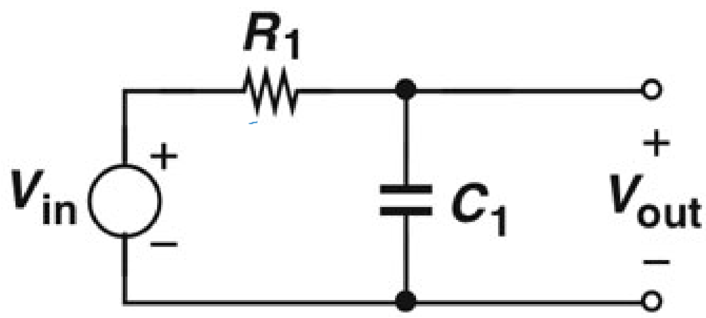
$$20\log\sqrt{2} \approx 3\,\text{dB}$$



-3dB bandwidth is equal to $1/(R_D C_L)$

Example 11.6

Explain the relationship between the frequency response and step response of the simple low-pass filter.



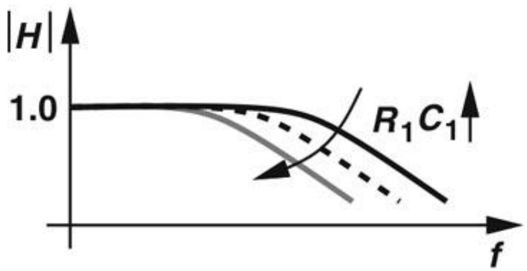
$$H(s)=\frac{V_{out}}{V_{in}}(s)=\frac{\frac{1}{C_1 s}}{\frac{1}{C_1 s}+R_1}=\frac{1}{R_1C_1 s+1}.$$

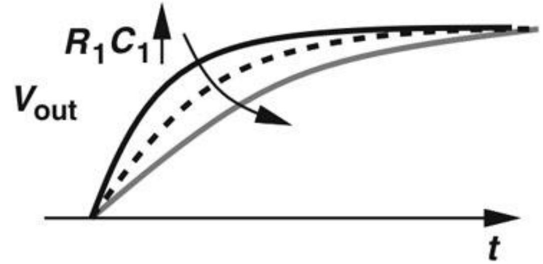
$$|H(s=j\omega)|=\frac{1}{\sqrt{R_1^2C_1^2\omega^2+1}}.$$

-3dB bandwidth = $1/R_1C_1$

Step response to $V_0u(t)$:

$$V_{out}(t)=V_0(1-\exp\frac{-t}{R_1C_1})u(t).$$





- **The relationship is such that as R_1C_1 increases, the bandwidth *drops* and the step response becomes *slower*.**

Bode's Rules

The task of obtaining $|H(j\omega)|$ from $H(s)$ and plotting the result is tedious. We often utilitze Bode's rules (approximation) to construct $|H(j\omega)|$ rapidly.

<Bode's rules>

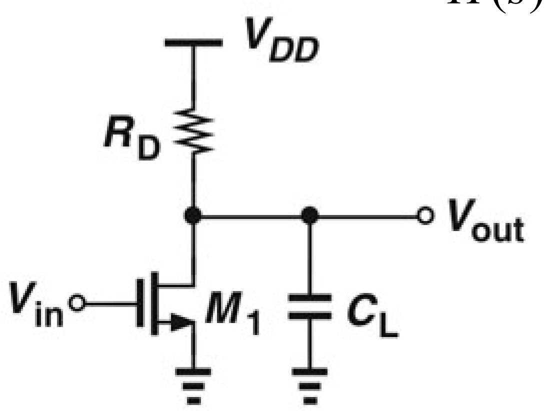
$$H(s) = A_0 \frac{\left(1+\frac{s}{\omega_{z1}}\right)\left(1+\frac{s}{\omega_{z2}}\right)\cdots}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)\cdots}$$

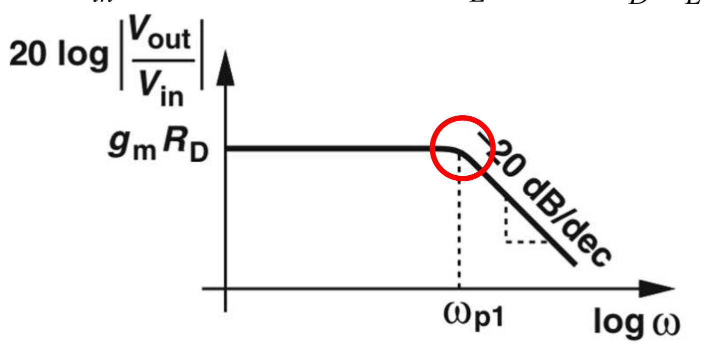
-As ω passes each pole frequency, the slope of $|H(j\omega)|$ decreases by 20 dB/dec.

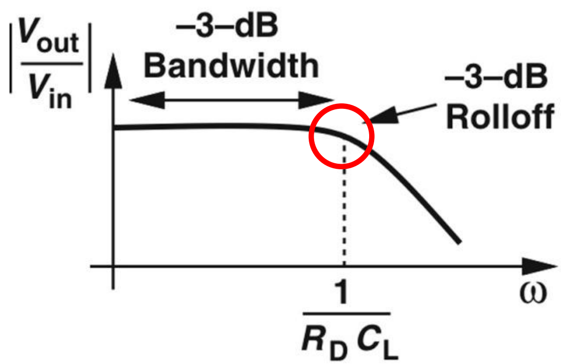
-As ω passes each zero frequency, the slope of $|H(j\omega)|$ increases by 20 dB/dec.

<Example 11.7> Construct the Bode plot of $|H(j\omega)|$ for the CS stage.

$$H(s) = \frac{V_{out}}{V_{in}}(s) = -g_m\left(R_D \,||\, \frac{1}{C_L s}\right) = \frac{-g_m R_D}{R_D C_L s + 1}. \qquad |\omega_{p1}| = \frac{1}{R_D C_L}.$$



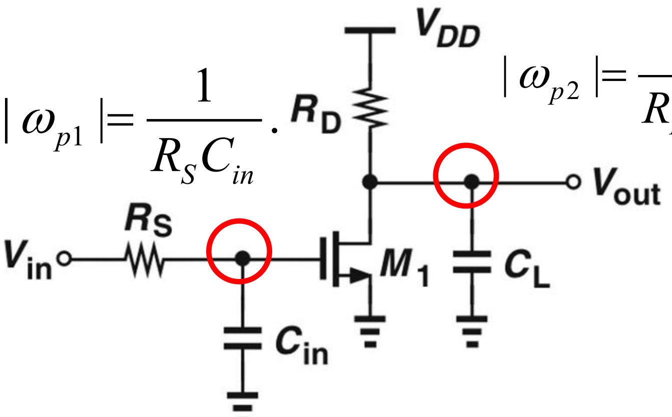




Identify the Poles (Speed Bottleneck)

Generalized from the CS stage: if node j in the signal path exhibits a small-signal resistance of R_j to ground and a capacitance of C_j to ground, then it contributes a pole of magnitude $(R_jC_j)^{-1}$ to the transfer function.

<Example 11.8> Determine the poles of the circuit. Assume λ=0.

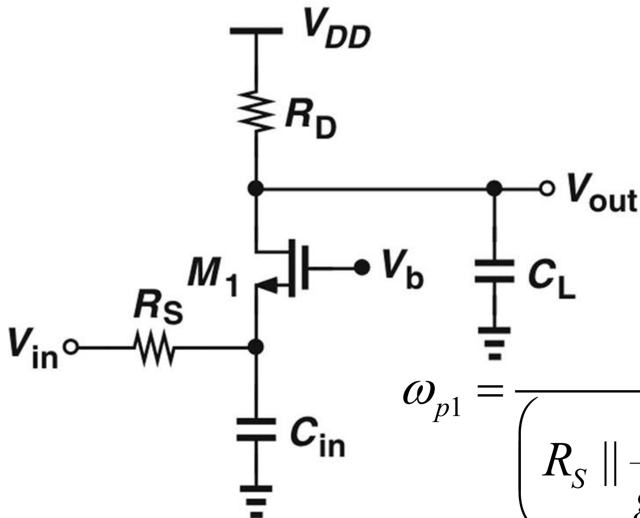


$$|\omega_{p1}| = \frac{1}{R_S C_{in}}.$$

$$|\omega_{p2}| = \frac{1}{R_D C_L}.$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{g_m R_D}{\sqrt{(1+\omega^2/\omega_{p1}^2)(1+\omega^2/\omega_{p2}^2)}}.$$

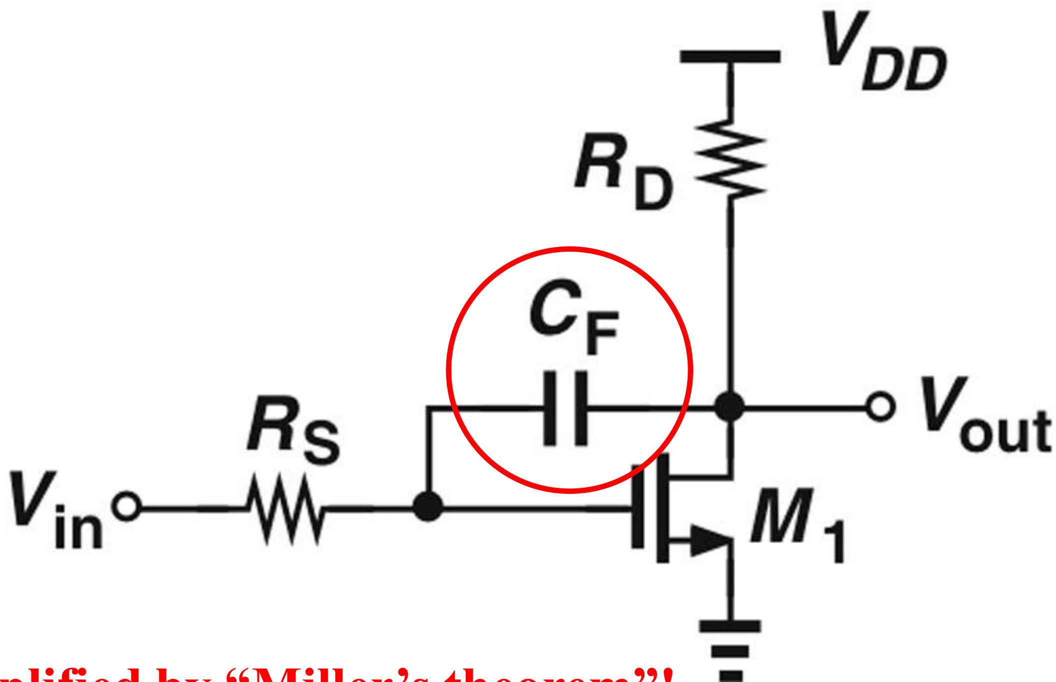
<Example 11.9> Determine the poles of the circuit. Assume λ=0.



$$\omega_{p1} = \frac{1}{\left(R_S \,||\, \frac{1}{g_m}\right) C_{in}}.$$

$$|\omega_{p2}| = \frac{1}{R_D C_L}$$

Circuit with Floating Capacitor



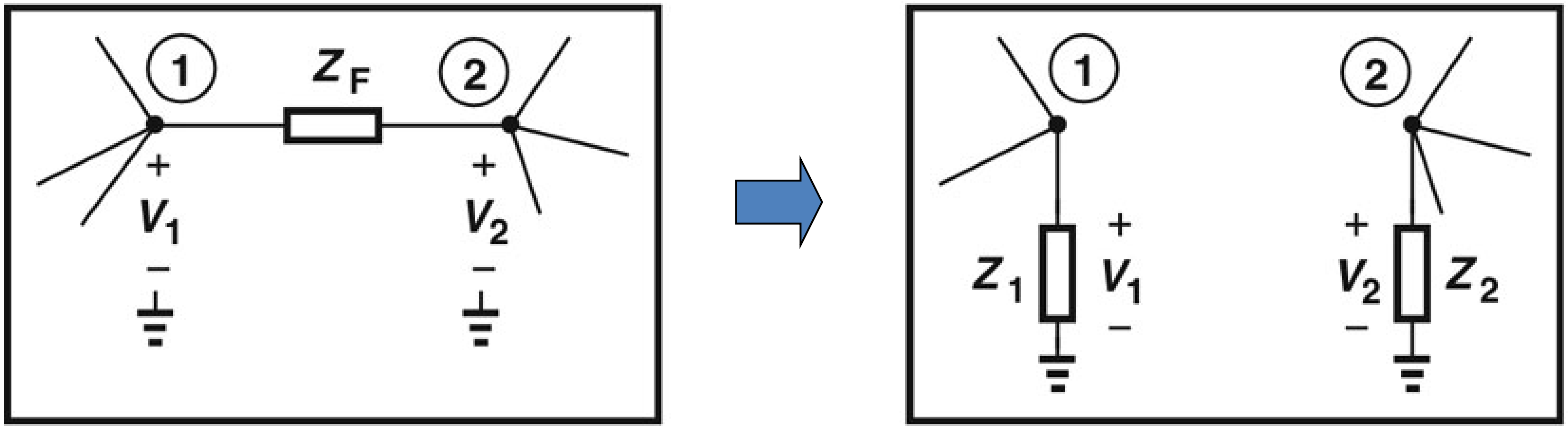
Can be simplified by "Miller's theorem"!

- **The pole of a circuit is computed by finding the effective resistance and capacitance from a node to GROUND.**
- **The circuit above creates a problem since neither terminal of C_F is grounded.**

Miller's Theorem

A method that "transforms" a floating capacitor to two grounded capacitors.

General circuit including a floating impedance | **Equivalent obtained from Miller's theorem**



<Two observations>

-The current drawn by Z_F from node 1 must be equal to that drawn by Z_1 $\qquad \dfrac{V_1 - V_2}{Z_F} = \dfrac{V_1}{Z_1}$

-The current injected to node 2 must be equal to that injected by Z_2 $\qquad \dfrac{V_1 - V_2}{Z_F} = -\dfrac{V_2}{Z_2}.$

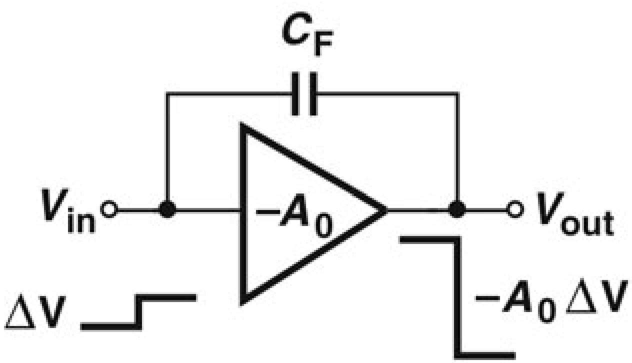
Denote the voltage gain from node 1 to node 2 by A_v

$$Z_1 = Z_F \frac{V_1}{V_1 - V_2} = \frac{Z_F}{1 - A_v} \qquad Z_2 = Z_F \frac{-V_2}{V_1 - V_2} = \frac{Z_F}{1 - \frac{1}{A_v}}.$$

The floating impedance is *reduced* by a factor of 1-A_v when "seen" at node 1

ZF as a Single Capacitor CF

Inverting amplifier with floating capacitor



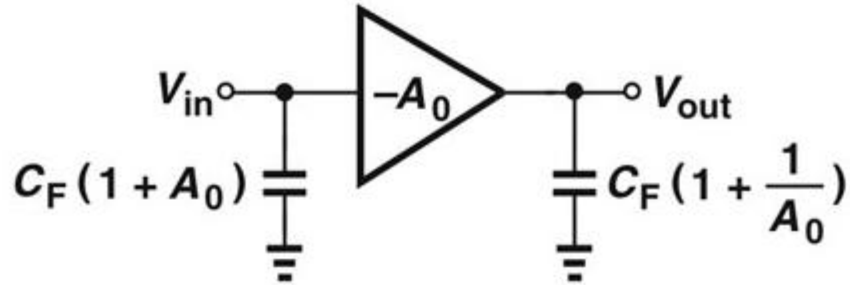
$$Z_1 = \frac{Z_F}{1-A_v} = \frac{1}{(1+A_0)C_F s},$$

Miller multiplication

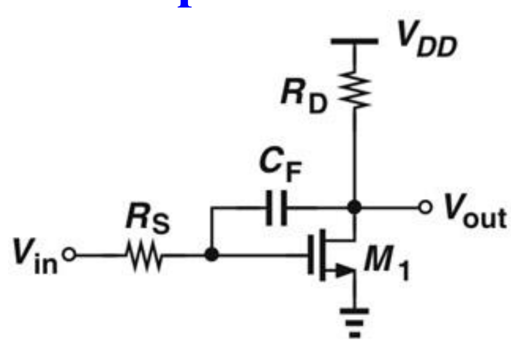
$$Z_2 = \frac{Z_F}{1-\frac{1}{A_v}} = \frac{1}{\left(1+\frac{1}{A_0}\right)C_F s}, \approx \frac{1}{C_F s}$$

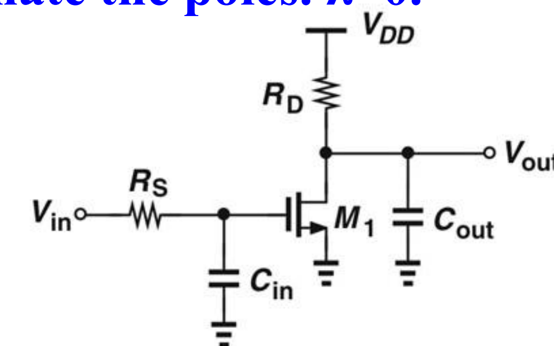
Miller effect!!

Equivalent circuit from Miller's theorem



<Example 11.10> Estimate the poles. λ=0.





$$C_{in} = (1+A_0)C_F = (1+g_m R_D)C_F$$

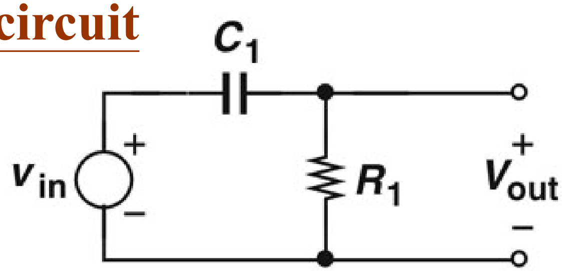
$$C_{out} = \left(1+\frac{1}{g_m R_D}\right)C_F,$$

$$\omega_{in} = \frac{1}{R_S C_{in}} = \frac{1}{R_S(1+g_m R_D)C_F}$$

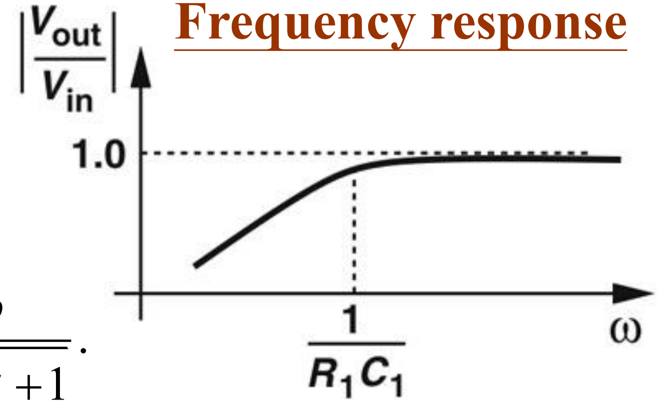
$$\omega_{out} = \frac{1}{R_D C_{out}} = \frac{1}{R_D\left(1+\frac{1}{g_m R_D}\right)C_F}.$$

High-Pass Filter

The circuit

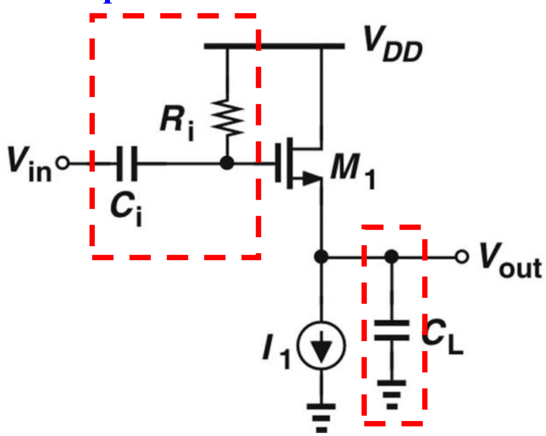


Frequency response



$$\frac{V_{out}}{V_{in}}(s) = \frac{R_1}{R_1 + \frac{1}{C_1 s}} = \frac{R_1C_1s}{R_1C_1s+1}, \Rightarrow \left|\frac{V_{out}}{V_{in}}\right| = \frac{R_1C_1\omega}{\sqrt{R_1^2C_1^2\omega_1^2+1}}.$$

<Example 11.11> Source follower in an audio amplifier. Assume λ=0, g_m = 1/(200Ω), and R_i = 100kΩ. Determine the minimum C_i and maximum C_L.



Audio signal component as low as 20Hz

Set the corner frequency $1/(R_iC_i)$ to 2π ×20Hz $\Rightarrow C_i = 79.6\text{nF}$.

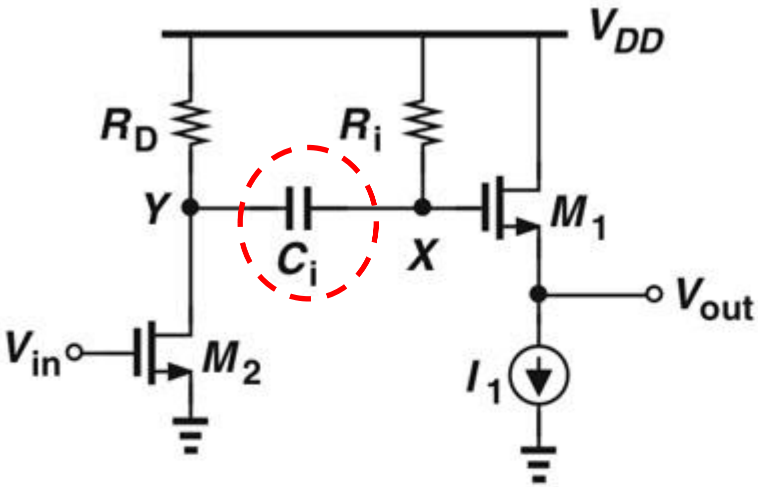
Set the pole frequency to upper end of audio 20kHz

$$\omega_{p,out} = \frac{g_m}{C_L} = 2\pi \times (20\text{kHz}), \Rightarrow C_L = 39.8\text{nF}.$$

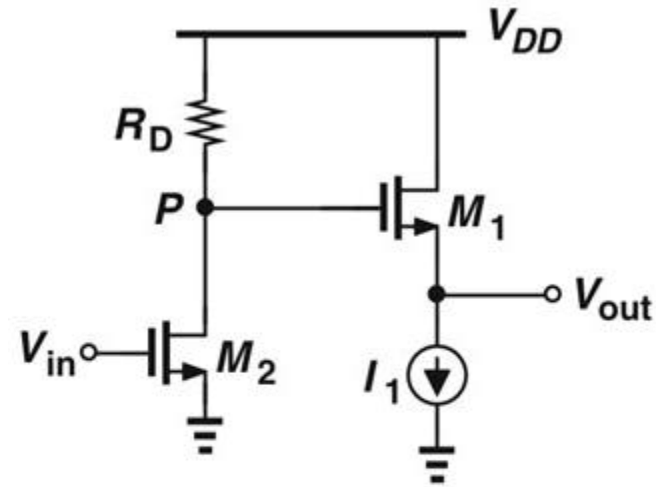
Why do we need C_i??

Capacitor Coupling

With coupling capacitor



Without coupling capacitor (direct coupling)

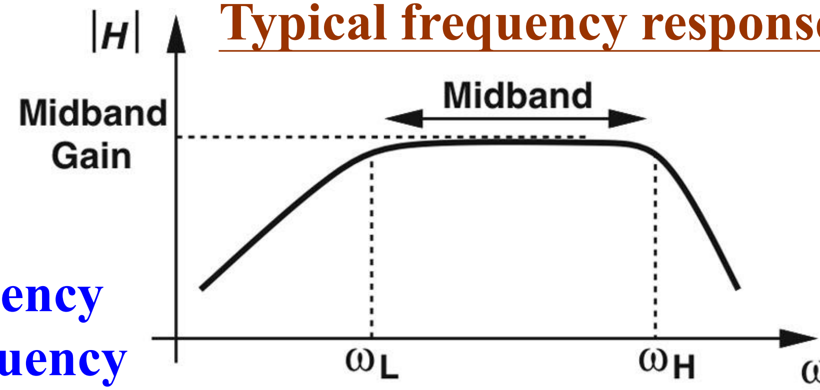


Maximize the CS voltage gain

Set $V_Y = V_{GS2} - V_{TH2}$ (why?) (200mV) ⟺ **$V_P = V_{GS1} + V_{I1}$ (600mV~700mV)**

$$A_v = -\sqrt{2\mu_n C_{ox}\frac{W}{L} I_D R_D},$$

Typical frequency response

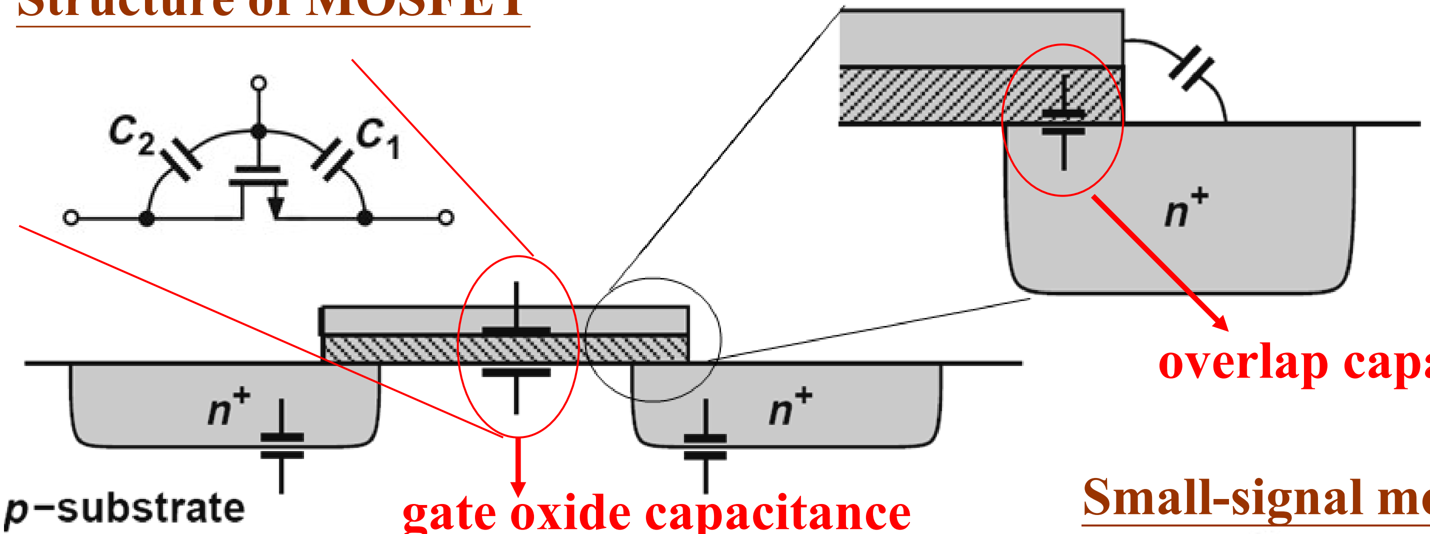


ω_L: lower corner, lower "cut-off" frequency

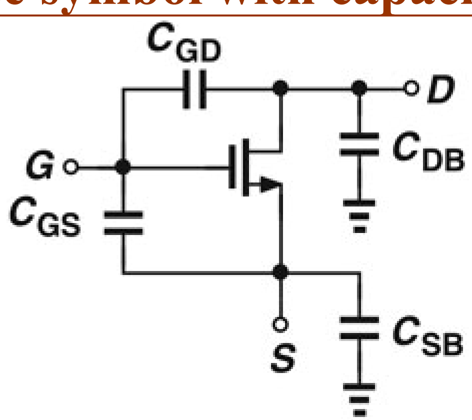
ω_H: upper corner, upper "cut-off" frequency

High-Frequency Model of MOSFET

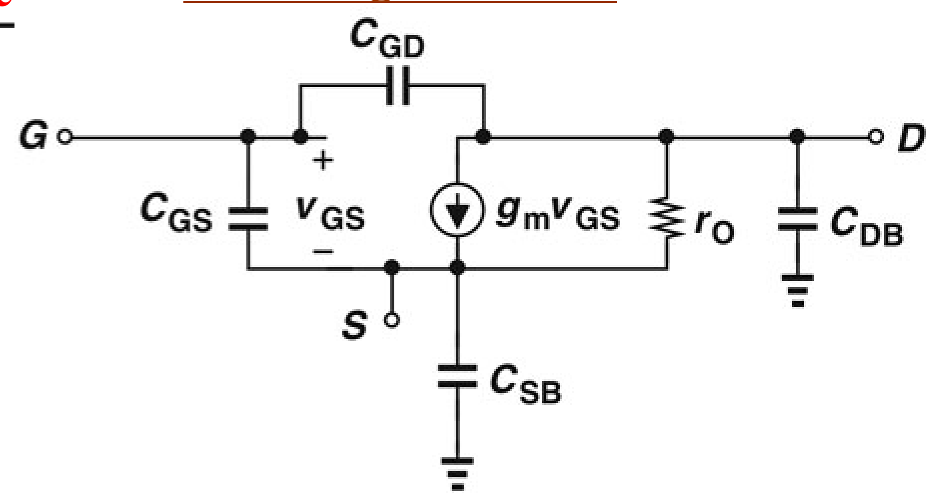
Structure of MOSFET



Device symbol with capacitances



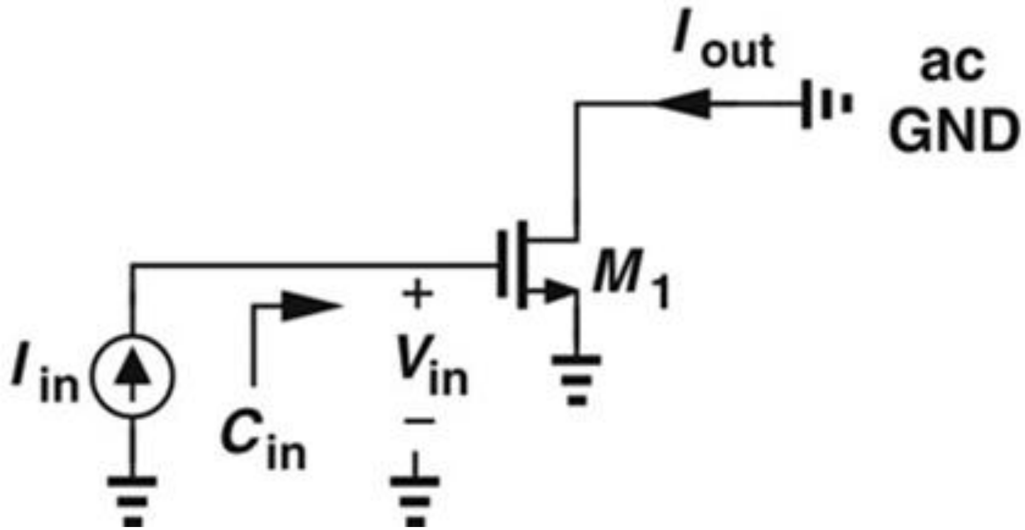
Small-signal model



Transit (Cut-Off) Frequency f_T

Define a quantity that represents the ultimate speed of the transistor

f_T: the frequency at which the small-signal *current gain* of the device falls to unity



$$Z_{in} = \frac{1}{sC_{GS}} \quad I_{out} = g_m I_{in} Z_{in} \quad \Rightarrow \quad \frac{I_{out}}{I_{in}} = \frac{g_m}{sC_{GS}}$$

$$\boxed{\omega_T = \frac{g_m}{C_{GS}}}$$

- **Transit frequency, f_T, is defined as the frequency where the current gain from input to output drops to 1.**

Concepts We Have Learned So Far

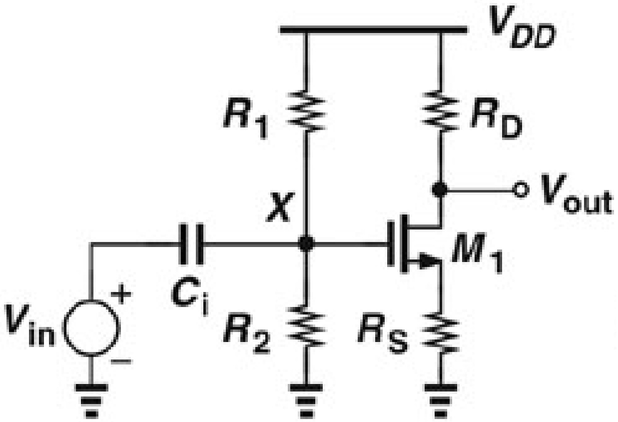
- **The frequency response refers to the magnitude of the transfer function of a system.**
- **Bode's approximation simplifies the task of plotting the frequency response if poles and zeros are known.**
- **In general, it is possible to associate a pole with each node in the signal path.**
- **Miller's theorem helps to decompose floating capacitors into grounded elements.**
- **MOS devices exhibit various capacitances that limit the speed of circuits.**

Analysis Procedure

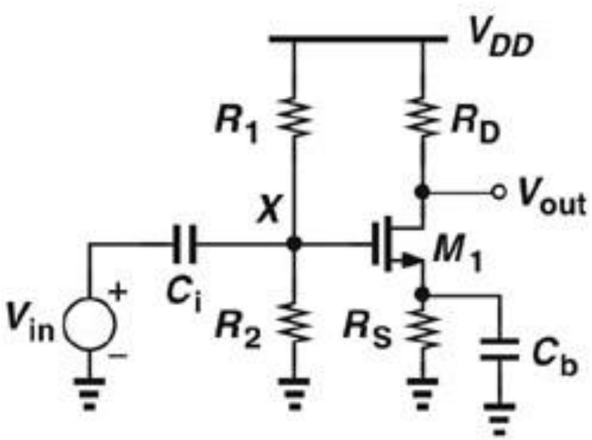
- **Determine which capacitor impact the low-frequency region of the response and calculate the low-frequency pole (neglect transistor capacitance).**
- **Calculate the midband gain by replacing the capacitors with short circuits (neglect transistor capacitance).**
- **Include transistor capacitances.**
- **Merge capacitors connected to AC grounds and omit those that play no role in the circuit.**
- **Determine the high-frequency poles and zeros.**
- **Plot the frequency response using Bode's rules or exact analysis.**

Low-Frequency Response of CS

General CS stage



Bypass capacitor added



Low frequency response

$$V_{out}/V_{in} = (V_{out}/V_X)(V_X/V_{in})$$

$$V_{out}/V_X = -R_D/(R_S + 1/g_m)$$

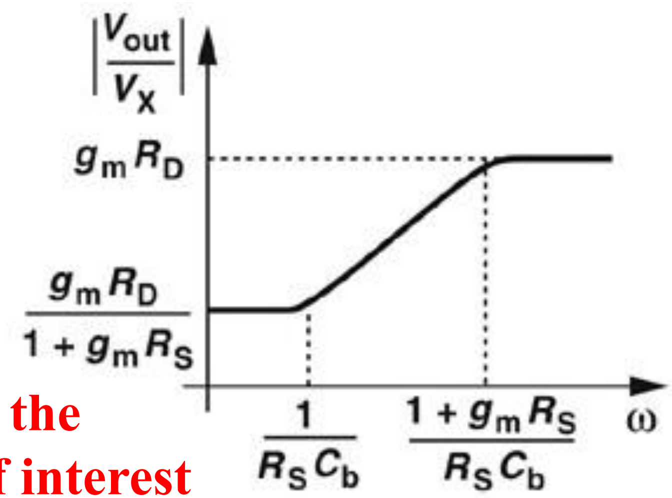
$$\frac{V_X}{V_{in}}(s) = \frac{R_1 \parallel R_2}{R_1 \parallel R_2 + \dfrac{1}{C_i s}} = \frac{(R_1 \parallel R_2)C_i s}{(R_1 \parallel R_2)C_i s + 1}.$$

$$\boxed{\frac{1}{2\pi[(R_1 \parallel R_2)C_i]} < f_{sig,min}.}$$

$$\frac{V_{out}}{V_X}(s) = \frac{-R_D}{R_S \parallel \dfrac{1}{C_b s} + \dfrac{1}{g_m}}$$

$$= \frac{-g_m R_D (R_S C_b s + 1)}{R_S C_b s + g_m R_S + 1}.$$

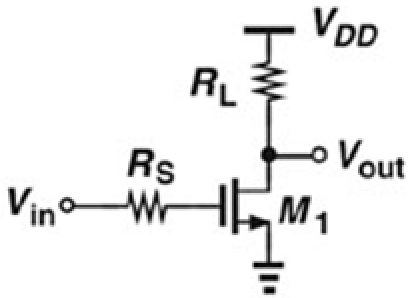
Bode plot



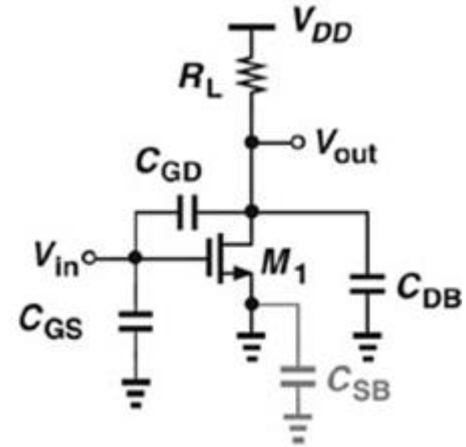
Pole needs to be chosen significantly smaller than the lowest signal frequency of interest

High-Frequency Model of CS

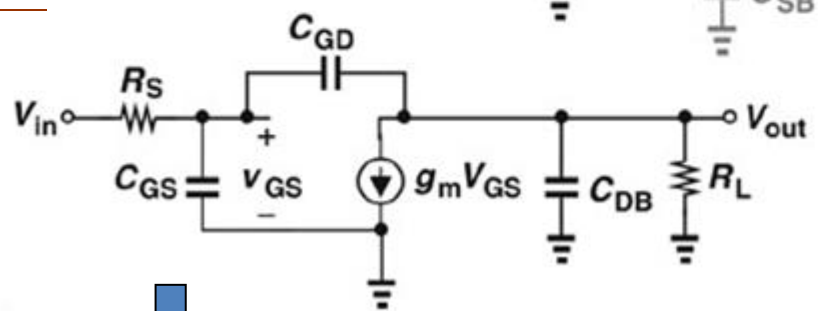
CS amplifier



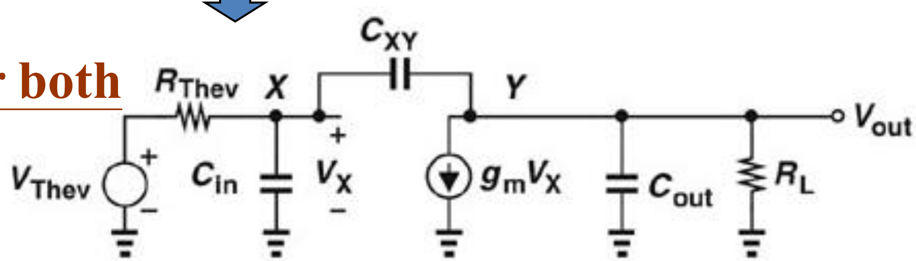
Transistor caps included



Small-signal equivalents



Unified for both

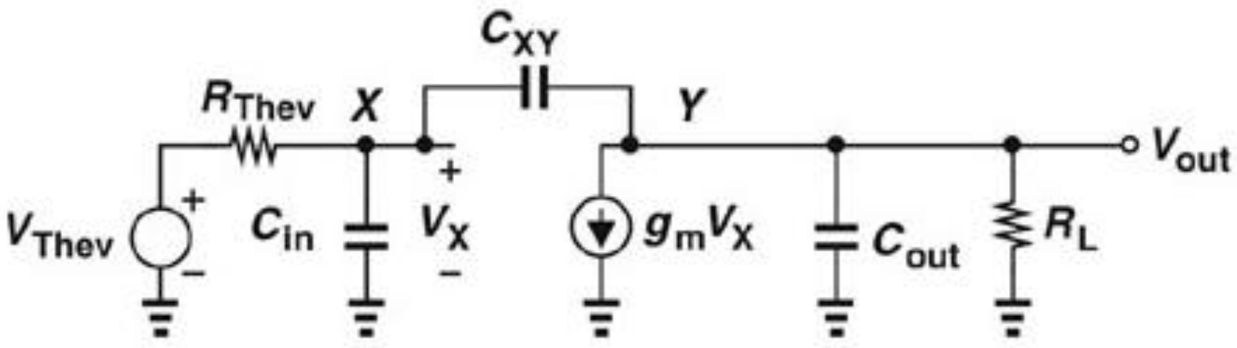


$R_S << r_\pi$

$R_{Thev} \approx R_S$

$r_o \parallel R_L$

Use of Miller's Theorem



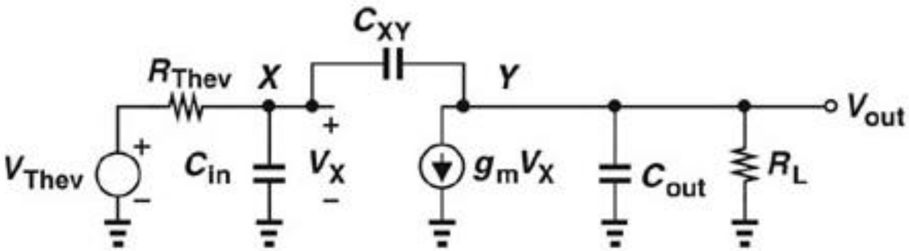
$$C_X = (1 + g_m R_L) C_{XY}$$

$$C_Y = \left(1 + \frac{1}{g_m R_L}\right) C_{XY}.$$

$$|\omega_{p,in}| = \frac{1}{R_{Thev}[C_{in} + (1 + g_m R_L) C_{XY}]}$$

$$|\omega_{p,out}| = \frac{1}{R_L\left[C_{out} + \left(1 + \frac{1}{g_m R_L}\right) C_{XY}\right]}.$$

Direct Analysis



$$\text{AtNode}X: (V_{out}-V_X)C_{XY}s = V_XC_{in}s + \frac{V_X - V_{Thev}}{R_{Thev}}$$

$$\text{AtNode}Y: (V_X-V_{out})C_{XY}s = g_mV_X + V_{out}\left(\frac{1}{R_L}+C_{out}s\right). \Rightarrow V_X = V_{out}\frac{C_{XY}s+\frac{1}{R_L}+C_{out}s}{C_{XY}s-g_m}$$

$$\Rightarrow V_{out}C_{XY}s - \left(C_{XY}s + C_{in}s + \frac{1}{R_{Thev}}\right)\frac{C_{XY}s+\frac{1}{R_L}+C_{out}s}{C_{XY}s-g_m}V_{out} = \frac{-V_{Thev}}{R_{Thev}}.$$

$$\Rightarrow \frac{V_{out}}{V_{Thev}}(s) = \frac{(C_{XY}s-g_m)R_L}{as^2+bs+1},$$

$$a = R_{Thev}R_L(C_{in}C_{XY}+C_{out}C_{XY}+C_{in}C_{out})$$

$$b = (1+g_mR_L)C_{XY}R_{Thev} + R_{Thev}C_{in} + R_L(C_{XY}+C_{out}).$$

More Analysis

$$\frac{V_{out}}{V_{Thev}}(s) = \frac{(C_{XY}s - g_m)R_L}{as^2 + bs + 1},$$

$$a = R_{Thev}R_L(C_{in}C_{XY} + C_{out}C_{XY} + C_{in}C_{out})$$

$$b = (1 + g_m R_L)C_{XY}R_{Thev} + R_{Thev}C_{in} + R_L(C_{XY} + C_{out}).$$

There is a zero

$$\omega_z = \frac{g_m}{C_{XY}}.$$

Miller's theorem fails to predict this zero!!

Usually at very high frequency, so…OK!

There are two poles

$$as^2 + bs + 1 = \left(\frac{s}{\omega_{p1}} + 1\right)\left(\frac{s}{\omega_{p2}} + 1\right) = \frac{s^2}{\omega_{p1}\omega_{p2}} + \left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right)s + 1.$$

Suppose $\omega_{p2} >> \omega_{p1}$ (dominant pole approximation) $b = \dfrac{1}{\omega_{p1}}$,

$$|\omega_{p1}| = \frac{1}{\underline{(1 + g_m R_L)C_{XY}R_{Thev} + R_{Thev}C_{in}} + \underline{R_L(C_{XY} + C_{out})}}.$$

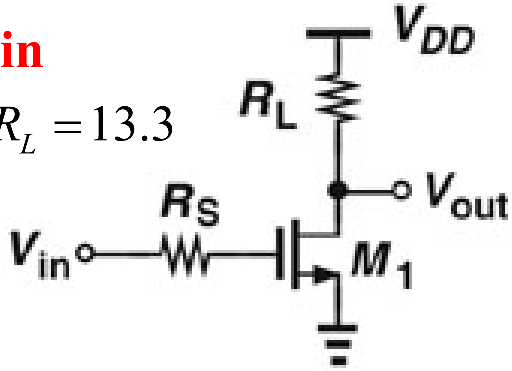
$$|\omega_{p2}| = \frac{b}{a} = \frac{(1 + g_m R_L)C_{XY}R_{Thev} + R_{Thev}C_{in} + R_L(C_{XY} + C_{out})}{R_{Thev}R_L(C_{in}C_{XY} + C_{out}C_{XY} + C_{in}C_{out})}.$$

Example 11.18

In the CS stage, R_S = 200Ω, C_{GS} = 250fF, C_{GD} = 80fF, C_{DB} = 100fF,, g_m = 1/(150Ω), λ=0, R_L = 2kΩ. Plot the frequency response with (a) Miller's approximation, (b) the exact transfer function, (c) the dominant-pole approximation.

Gain

$g_m R_L = 13.3$



(a)

$$|\omega_{p,in}| = 2\pi \times (571\text{MHz})$$
$$|\omega_{p,out}| = 2\pi \times (428\text{MHz}).$$

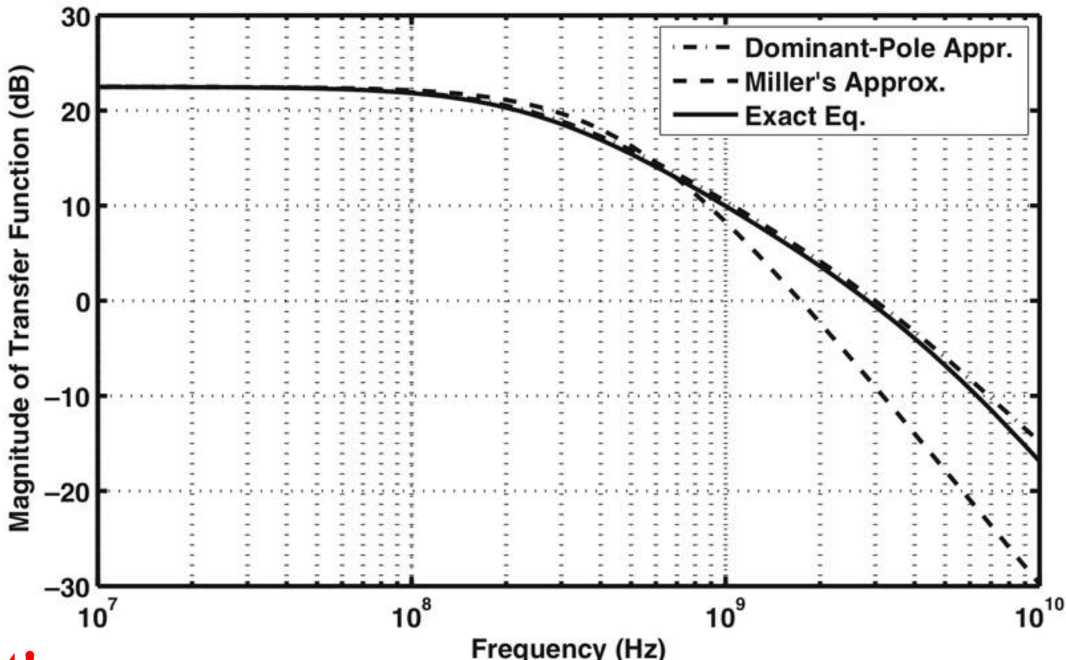
(b) zero: $g_m / C_{GD} = 2\pi \times (13.3\text{ GHz})$

a = 2.12 × 10^{-20}s^{-2}, b = 6.39 × 10^{-10}s

$$|\omega_{p1}| = 2\pi \times (264\text{MHz})$$
$$|\omega_{p2}| = 2\pi \times (4.53\text{GHz}).$$

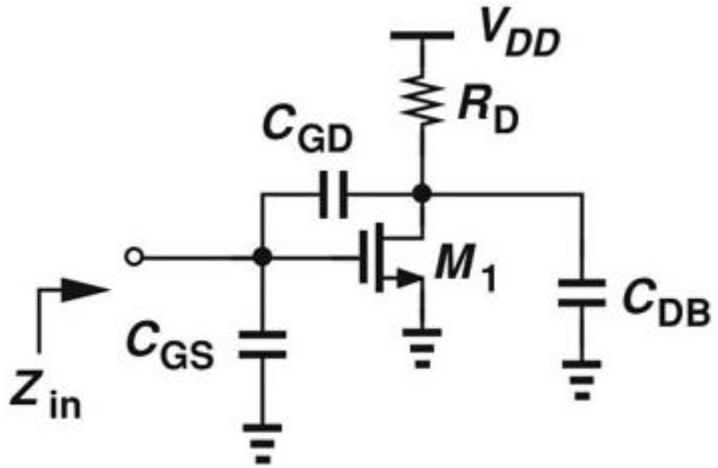
(c)

$$|\omega_{p1}| = 2\pi \times (249\text{MHz})$$
$$|\omega_{p2}| = 2\pi \times (4.79\text{GHz}).$$



Very different!

Input Impedance

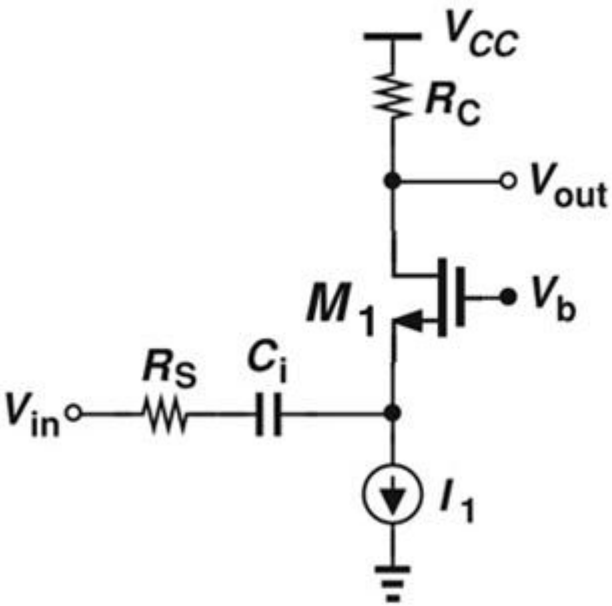


$$Z_{in} \approx \frac{1}{[C_{GS} + (1 + g_m R_D) C_{GD}]s}.$$

- **With a high voltage gain, the Miller effect may substantially lower the input impedance at high frequencies**

Frequency Response of CG

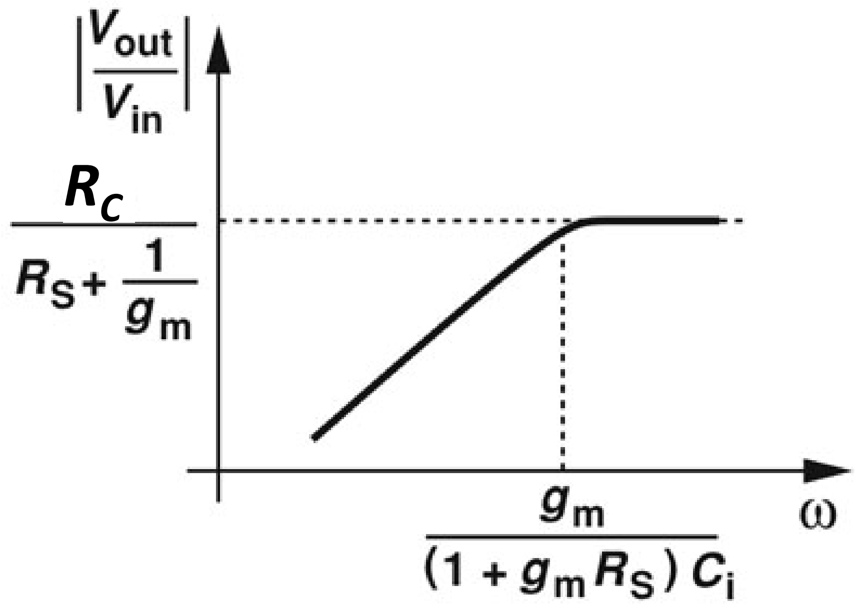
General CG stage



$$|\omega_p| = \frac{g_m}{(1+g_m R_S)C_i}$$

$$\frac{1}{C_i s} << R_S + \frac{1}{g_m}$$

Frequency response



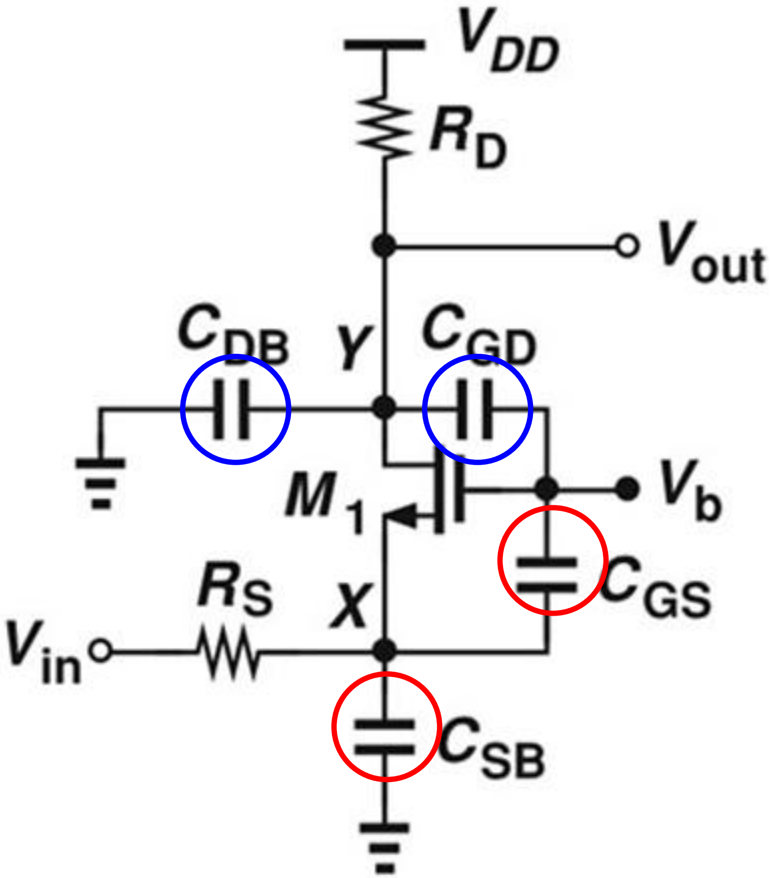
Midband gain: $R_C/(R_S + 1/g_m)$ **Slide 3-31**

Replace R_S with $R_S + (C_i s)^{-1}$

$$\frac{V_{out}}{V_{in}}(s) = \frac{R_C}{R_S + (C_i s)^{-1} + 1/g_m} = \frac{g_m R_C C_i s}{(1+g_m R_S)C_i s + g_m}.$$

- **Capacitive coupling leads to low-frequency roll-off**

Frequency Response of CG



$$|\omega_{p,X}| = \frac{1}{\left(R_S \parallel \frac{1}{g_m}\right) C_X}, \approx \omega_T$$

$$|\omega_{p,Y}| = \frac{1}{R_D C_Y},$$

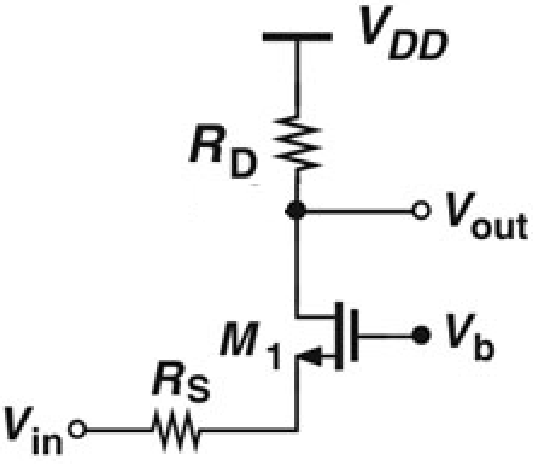
Node X (C_X): C_{SB}+C_{GS} to ground

Node Y (C_Y): C_{DB}+C_{GD} to ground

No capacitance between input and output: No Miller effect.

Example 11.20

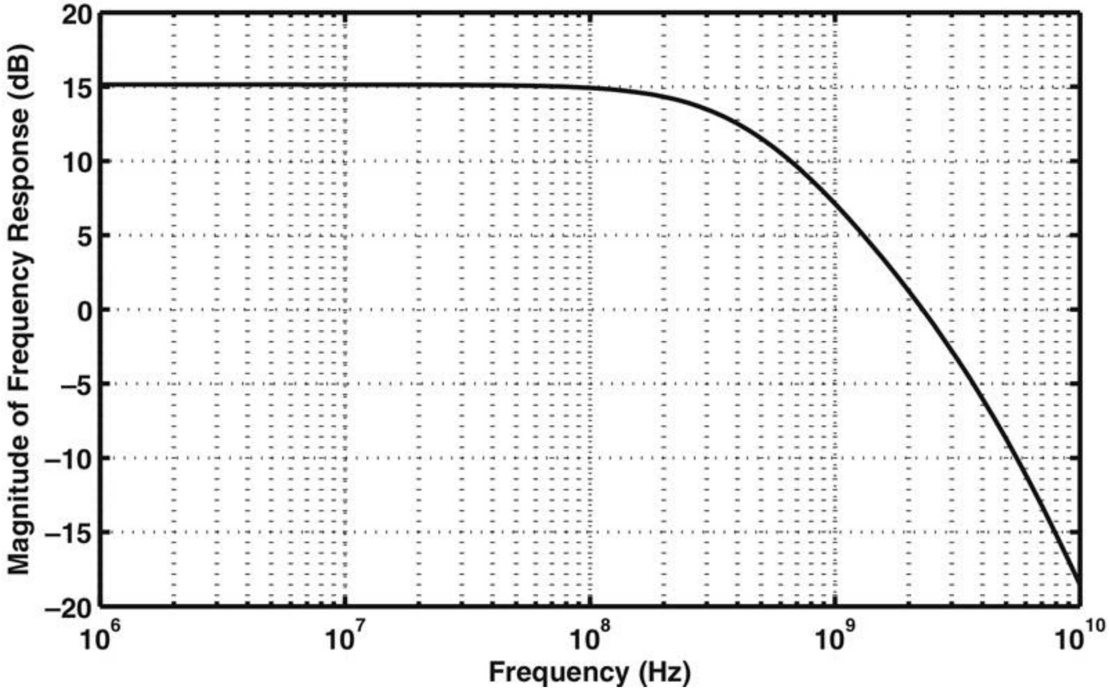
Plot the frequency response of the CG stage using 11.18 parameters.



$$|\omega_{p,X}| = \frac{1}{\left(R_S \parallel \frac{1}{g_m}\right) C_X}, \quad |\omega_{p,Y}| = \frac{1}{R_D C_Y},$$

Limitation

$$|\omega_{p,X}| = 2\pi \times (5.31\text{GHz}) \quad \boxed{|\omega_{p,Y}| = 2\pi \times (442\text{MHz}).}$$

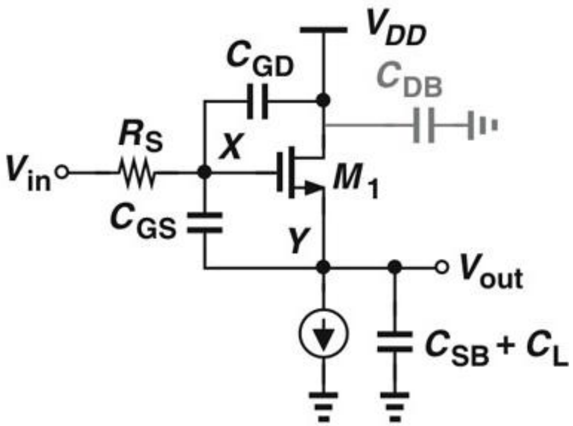


Gain = $R_D/(R_S + 1/g_m)$

= 5.7 < CS =13.3

Frequency Response of SF

Source follower



$$\text{Node } X: \frac{v_{out}+v_{GS}-v_{in}}{R_S}+(v_{out}+v_{GS})C_{GD}s+v_{GS}C_{GS}s=0$$

$$\text{Output node}: v_{GS}C_{GS}s+g_m v_{GS}=v_{out}(C_{SB}+C_L)s$$

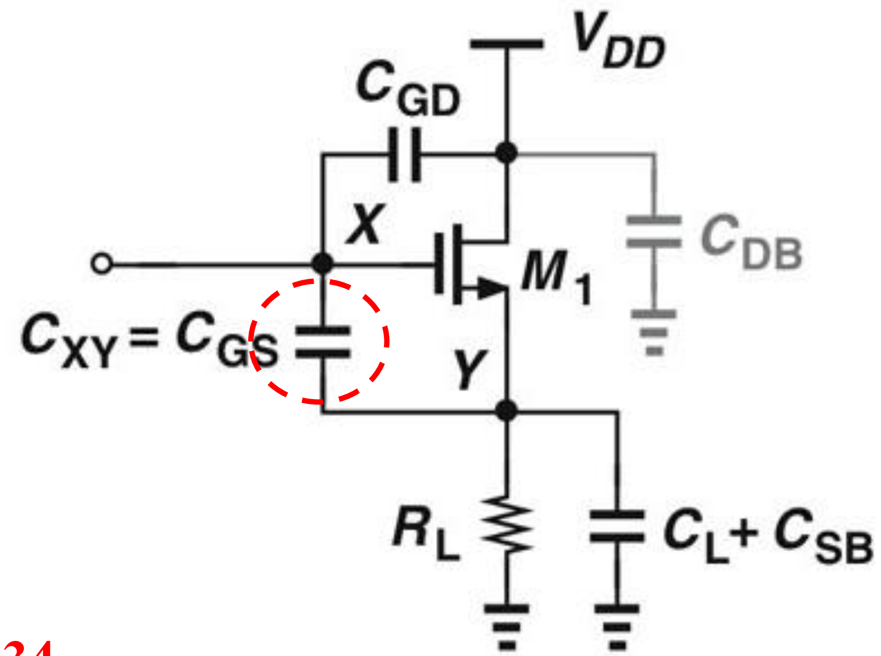
$$v_{GS}=\frac{v_{out}(C_{SB}+C_L)s}{C_{GS}s+g_m}$$

$$\frac{V_{out}}{V_{in}}=\frac{1+\dfrac{C_{GS}}{g_m}s}{as^2+bs+1},$$

$$a=\frac{R_S}{g_m}[C_{GD}C_{GS}+C_{GD}(C_{SB}+C_L)+C_{GS}(C_{SB}+C_L)]$$

$$b=R_SC_{GD}+\frac{C_{GD}+C_{SB}+C_L}{g_m}.$$

Input Impedances of SF



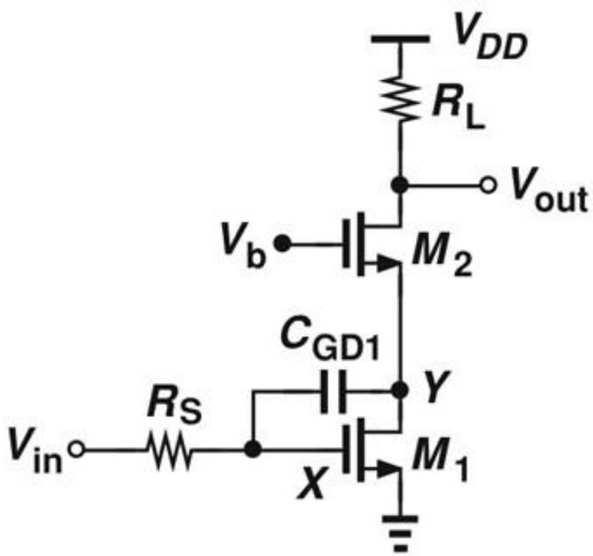
Miller's theorem!!

Slide 3-34

$$A_v = \frac{R_L}{R_L + \frac{1}{g_m}}, \qquad C_X = (1 - A_v)C_{XY} = \frac{1}{1 + g_m R_L} C_{XY}.$$

$$\boxed{C_{in} = C_{GD} + \frac{C_{GS}}{1 + g_m R_L}}$$

Poles of MOSFET Cascode

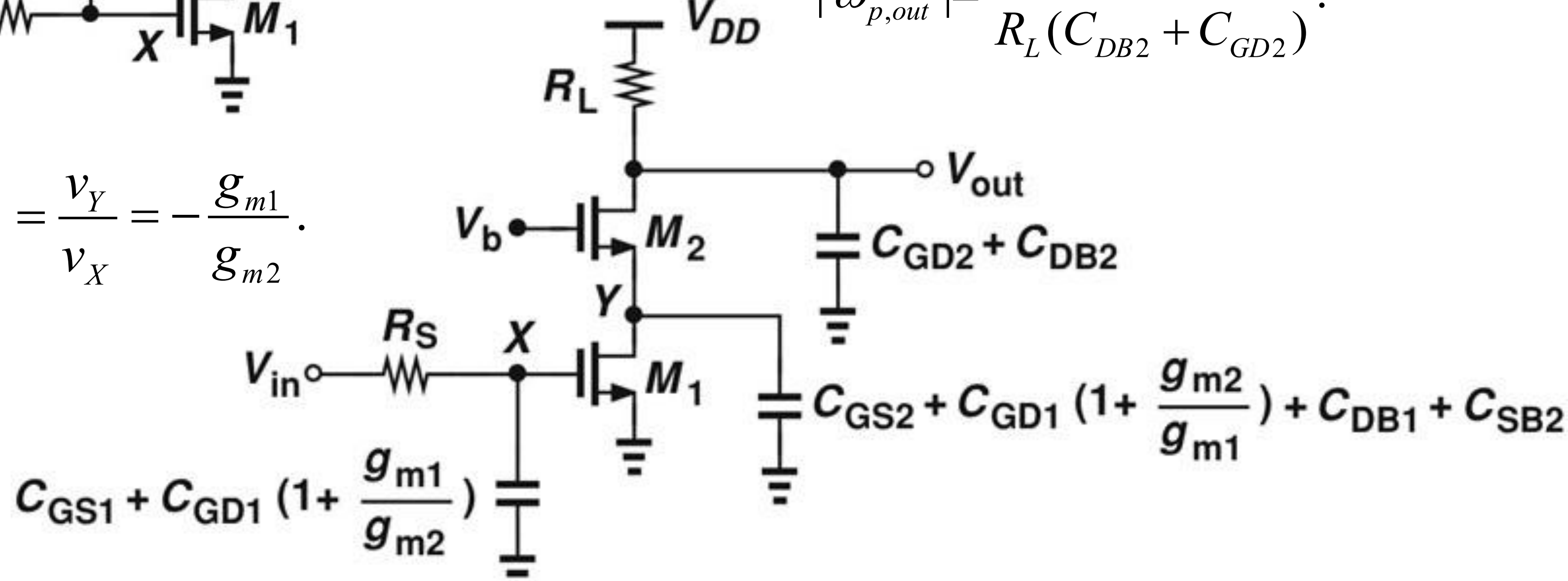


$$|\omega_{p,X}| = \frac{1}{R_S[C_{GS1} + \left(1 + \frac{g_{m1}}{g_{m2}}\right)C_{GD1}]}$$

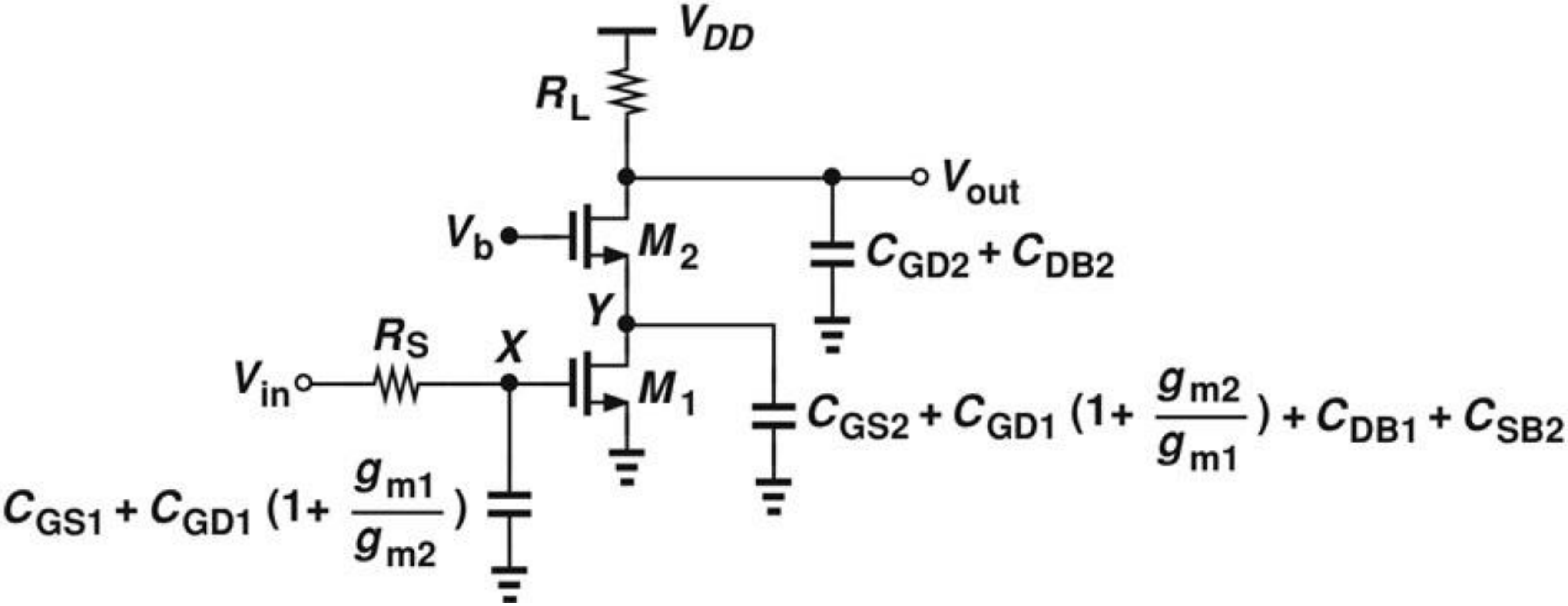
$$|\omega_{p,Y}| = \frac{1}{\frac{1}{g_{m2}}[C_{DB1} + C_{GS2} + \left(1 + \frac{g_{m2}}{g_{m1}}\right)C_{GD1} + C_{SB2}]}$$

$$|\omega_{p,out}| = \frac{1}{R_L(C_{DB2} + C_{GD2})}.$$

$$A_{v,XY} = \frac{v_Y}{v_X} = -\frac{g_{m1}}{g_{m2}}.$$

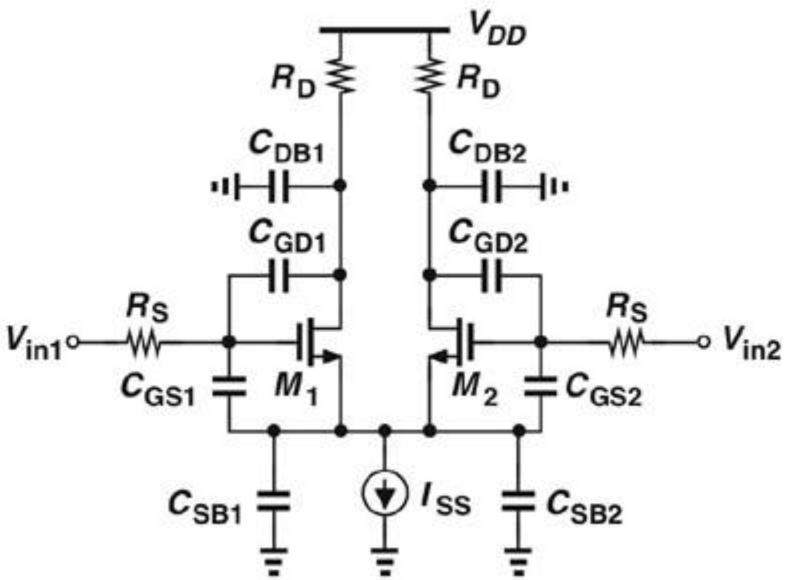


Input and Output Impedances

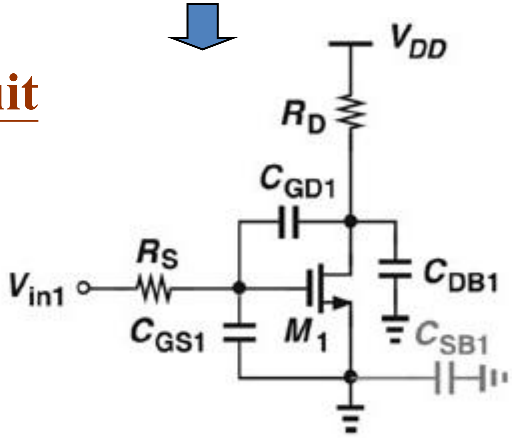


$$Z_{in} = \frac{1}{[C_{GS1} + \left(1 + \frac{g_{m1}}{g_{m2}}\right) C_{GD1}]s} \qquad Z_{out} = \frac{1}{s(C_{GD2} + C_{DB2})} \ || \ R_L$$

Frequency Response of DP



Half circuit

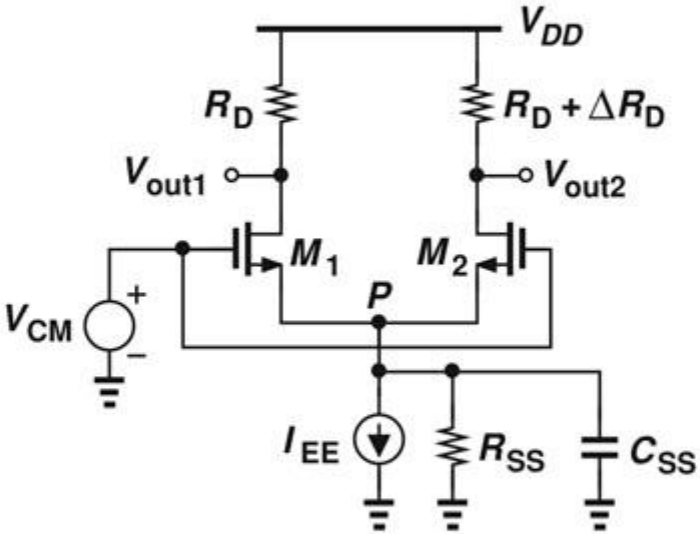


Slide 5-23

$$\frac{V_{out}}{V_{Thev}}(s) = \frac{(C_{XY}s - g_m)R_L}{as^2 + bs + 1},$$

Same as CS!

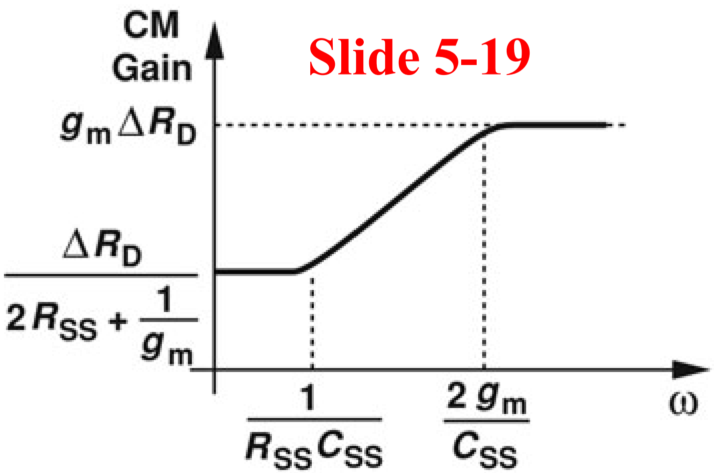
Common-Mode Frequency Response



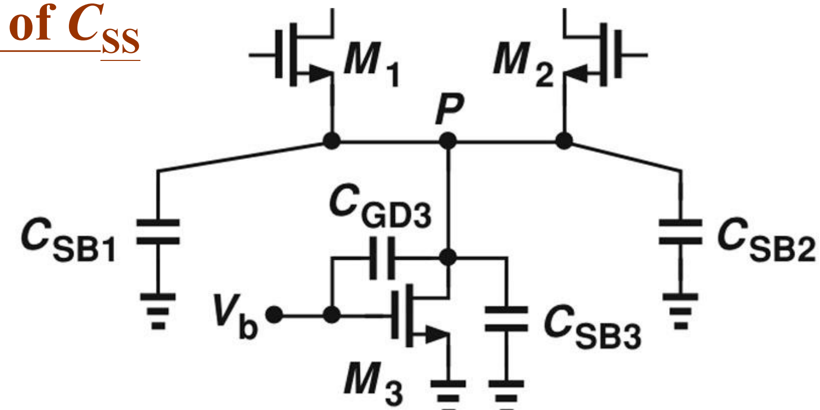
$$\left|\frac{\Delta V_{out}}{\Delta V_{CM}}\right| = \frac{\Delta R_D}{\dfrac{1}{g_m} + 2(R_{SS} \,\|\, \dfrac{1}{C_{SS}s})} = \frac{g_m \Delta R_D (R_{SS}C_{SS}+1)}{R_{SS}C_{SS}s + 2g_m R_{SS} + 1}.$$

$$2g_m R_{SS} >> 1 \qquad |\omega_z| = \frac{1}{R_{SS}C_{SS}} \qquad |\omega_p| = \frac{2g_m}{C_{SS}},$$

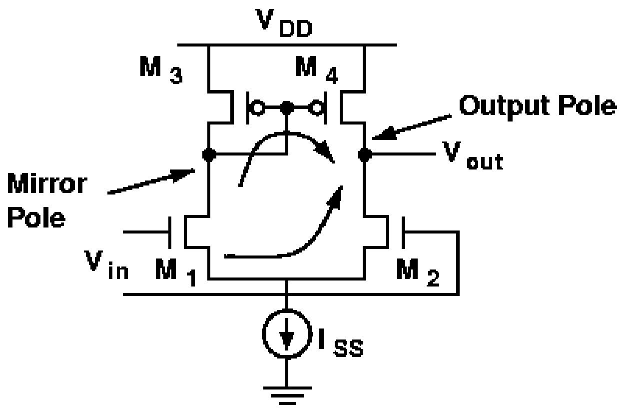
- C_{ss} **will lower the total impedance between point P to ground at high frequency, leading to higher CM gain which degrades the CM rejection ratio.**

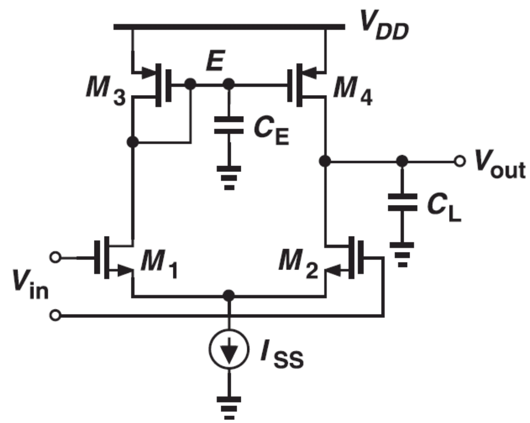


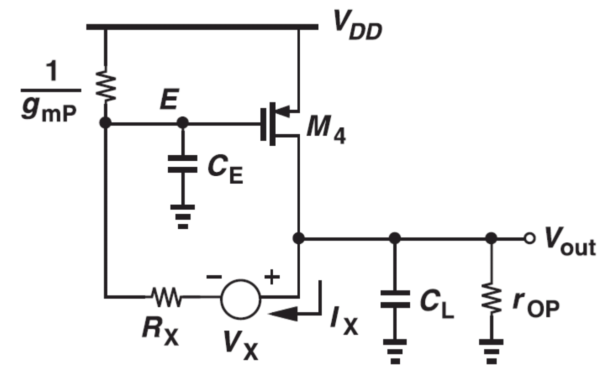
Source of C_{SS}



DP with Active Current Mirror







- This topology contains two signal paths with different transfer functions.
- Pole at node *E (Mirror pole)*

$$\omega_{pE}=\frac{g_{m3}}{C_E},\quad C_E\approx C_{GS3}+C_{GS4}+C_{DB3}+C_{DB1}+2C_{GD1}+g_{m4}\left(r_{O4}\parallel r_{O2}\right)C_{GD4}$$

$$V_X=g_{mN}r_{ON}V_{in},\ R_X=2r_{ON},\ V_E=\left(V_{out}-V_X\right)\frac{\left(C_Es+g_{mP}\right)^{-1}}{\left(C_Es+g_{mP}\right)^{-1}+R_X},\quad -g_{m4}V_E-I_X=V_{out}\left(sC_L+r_{OP}^{-1}\right)$$

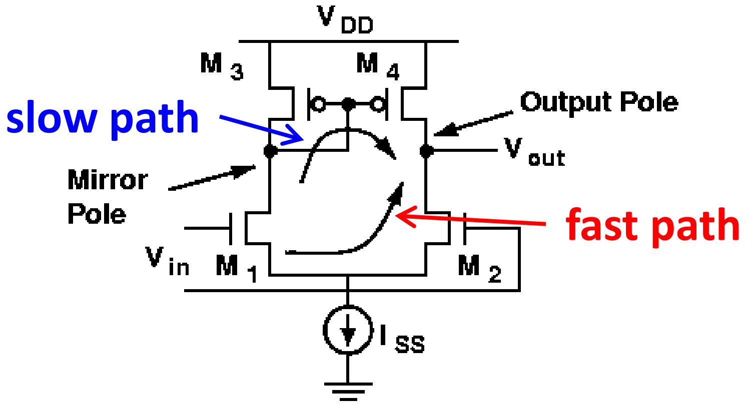
$$\frac{V_{out}}{V_{in}}=\frac{g_{mN}r_{ON}r_{OP}(2g_{mP}+sC_E)}{2r_{OP}r_{ON}C_EC_Ls^2+\left[\left(2r_{ON}+r_{OP}\right)C_E+r_{OP}\left(1+2g_{mP}r_{ON}\right)C_L\right]s+2g_{mp}\left(r_{ON}+r_{OP}\right)}$$

$$\omega_{P1}\approx\frac{2g_{mP}\left(r_{ON}+r_{OP}\right)}{\left(2r_{ON}+r_{OP}\right)C_E+r_{OP}(1+2g_{mp}r_{ON})C_L}\approx\frac{1}{\left(r_{ON}\parallel r_{OP}\right)C_L}\quad \text{if}\quad 2g_{mP}r_{ON}>>1\qquad \omega_{p2}\approx\frac{g_{mP}}{C_E}$$

DP with Active Current Mirror

- To find zero
  - Consider that the circuit consists of a "slow path" (M_1, M_3, and M_4) in parallel with a "fast path" (M_1, and M_2)
  - Slow path

$$A_{v1} = \frac{A_0}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)}$$



  - Fast path

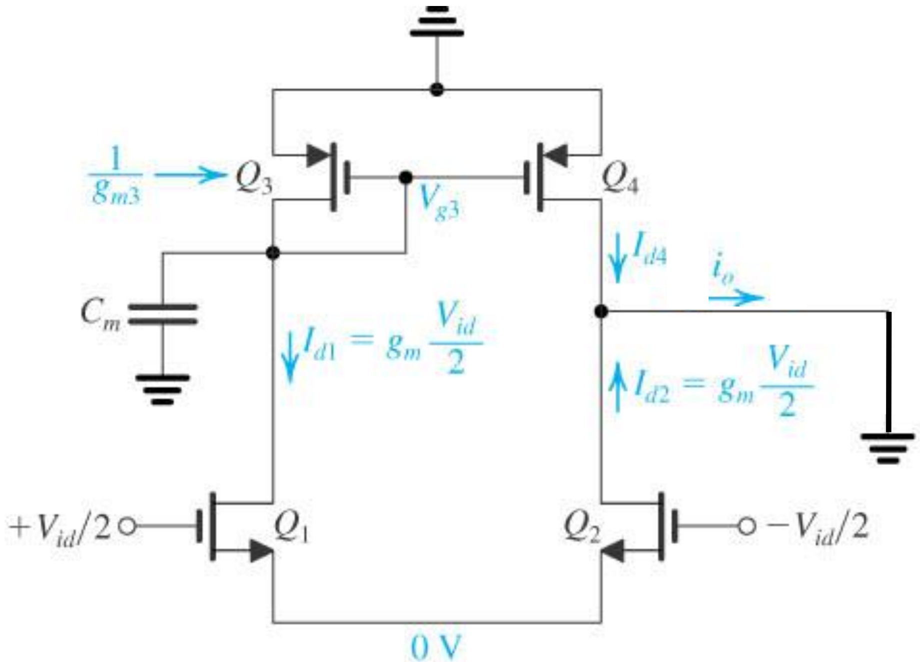
$$A_{v2} = \frac{A_0}{\left(1+\frac{s}{p_1}\right)}$$

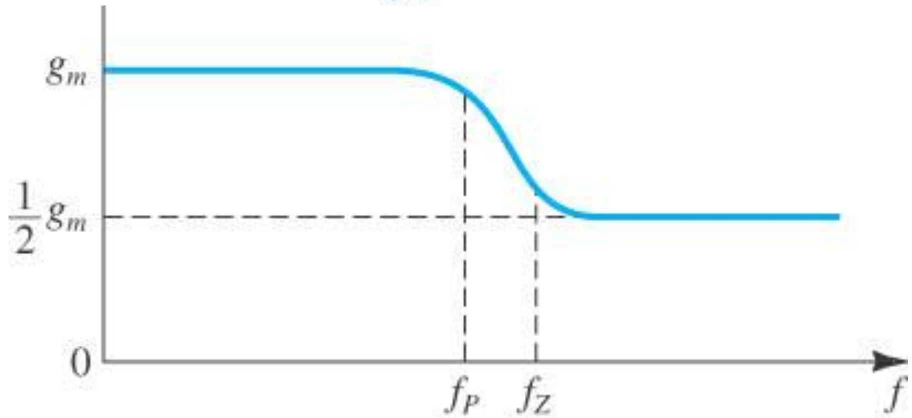
  - We have

$$\frac{V_{out}}{V_{in}} = A_{v1} + A_{v2} = \frac{A_0}{1+\frac{s}{p_1}}\left(1+\frac{1}{1+\frac{s}{p_2}}\right) = \frac{A_0\left(2+\frac{s}{p_2}\right)}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)} \quad \Rightarrow \quad z = 2p_2$$

  - Fully differential pair has no mirror pole, which is better than differential-to-single ended amp.

G_m Frequency Response





At low frequency: C_m is OPEN

$$i_{d4} = i_{d1} = g_m \frac{V_{id}}{2}, i_{d2} = g_m \frac{V_{id}}{2}$$

$$i_o = i_{d4} + i_{d2} = g_m V_{id}$$

$$G_m = i_o / V_{id} = g_m$$

At high frequency: C_m is SHORT

$$V_{g3} = 0, i_{d4} = 0, i_o = i_2 = g_m \frac{V_{id}}{2}$$

$$G_m = i_o / V_{id} = g_m / 2$$

G_m transfer function

$$G_m \equiv \frac{i_o}{V_{id}} = g_m \left(\frac{1 + sC_m / 2g_{m3}}{1 + sC_m / g_{m3}}\right)$$

$$f_P = \frac{g_{m3}}{2\pi C_m}, f_Z = \frac{2g_{m3}}{2\pi C_m}$$